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# Optimization of TV Advertising Plan Efficiency

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## Abstract

We present a new approach to generation of media plans of TV advertising campaigns. This approach is based on raw data obtained by television audience measurement systems. It provides that as many as possible target viewers are exposed to TV commercial proper number of times, satisfying additional constraints for budget and daily ratings. We use goal programming for modeling several criteria. Iteratively solving mixed integer problems we obtain a list of time slots in which TV commercial should be aired, i.e. media plan.

**Keywords:** TV media planning, mixed integer programming, goal programming

## 1 Introduction

TV advertising is the most significant channel of communication between companies and audience, providing the fastest propagation of advertising message. In last 50 years, trading of television advertising time has grown up into a multi-million dollars industry in all countries, regardless of level of its development. Consequently, optimization of TV advertising efficiency has been studied from both commercial and scientific aspects. The necessity for improvement of advertising efficiency in general, was expressed one century ago by John Wanamaker in his statement: “Half the money I spend on advertising is wasted; the trouble is I don’t know which half” [Iyer et al. 2005].

Advertising efficiency is expressed by ratio of the increment of revenue caused by advertising campaign, and advertising cost. However, revenue depends on many psychological factors that influence consumers desire to buy an advertised product or service [Belenky 2001]. These factors have to be taken into account

when advertising strategy is chosen. Advertising strategy usually considers intensity and dynamics of exposure to advertising message. It has to determine who are the target consumers of the product and what are their characteristics.

The main participants in the process of TV advertising are: advertiser, advertising agency, TV stations and the audience of viewers – potential consumers of the product. In this paper our intention is to maximize the efficiency of advertising campaign. We influence it by deciding in which commercial break (i.e. time slot) a TV commercial (hereinafter: TVC) will be aired. The selection of time slots in which TVC will be placed is called media plan and it is usually made in advertising agency by media planner. Although many researches on this issue have been done, in practice media planners often rely on their cognitive capabilities, skill and experience. The main goal of this paper is to show to media planners how more efficient plans can be constructed such that they generate growth of the advertised product consumption for less costs, taking into account all the participants demands and constraints.

In Section 2 an overview of previous researches in this field is given. Section 3 contains a brief description of the problem. In Section 4 the notation and mathematical model are introduced. Results of numerical experiments and their comparison to solution made in traditional way are given in Section 5. In Section 6 we give several concluding remarks.

## 2 Literature review

Problem of TV advertising has been studied using various approaches and assumptions.

Charnes et al. [Charnes et al. 1968] observed the impact of message frequencies on the percentage of target segment which is exposed, and expressed it as a log-normal distribution. Their goal programming model includes constraints related to the percentage of audience segment which must be exposed to TVC at least a given number of times.

Bollapragada and Garbiras [Bollapra and Garbiras 2004] observed the problem of reordering the initial advertisement schedule to achieve objectives of contracts agreed upon clients and broadcast television. Their main goal was to avoid the competing products conflict. The second one was to minimize perturbation of initial positions in slot.

Mihiotis and Tsakiris [Mihiotis and Tsakiris 2004] used a mathematical programming approach to determine the combination of time slots in which TVC will be inserted. The total sum of gross rating points is maximized subject to the limitation of the available budget.

Bollapragada, Bussieck and Mallik [Bollapragada et al. 2004] analyzed the problem of scheduling on broadcast television. They used mixed-integer programming to obtain a schedule which provides as evenly spaced airings of TVCs as possible.

Cetin and Esen [Cetin and Esen 2006] proposed an integer nonlinear programming model based on the weapon-target assignment problem, for solving

media allocation problem including budget allocation.

Saha, Pal and Pal [Saha et al. 2007] used graph coloring approach to model the problem of time slots selection such that the number of viewers of selected time slots is maximal.

In [Bhattacharya 2009] chance goal programming model was developed to solve the problem of selecting optimal number of advertisement in different media (newspapers and TV channels) within the available budget in order to maximize the sum of aggregated target rating points.

Jha, Aggarwal and Gupta [Jha et al. 2011] were investigating the optimal allocation of advertising budget to multiple products, advertised through different media (newspapers and websites) in a segmented market. The problem of finding the number of advertisements in each of media in order to maximize the desirable reach has been formulated as a multi-objective problem and solved using goal programming.

In [Stanojević et al. 2011] authors suggest a mathematical model which provides that as many as possible target viewers are exposed to TV commercial predefined number of times, satisfying additional constraints for budget and daily ratings. Campaign period is divided into sub-periods which are optimized iteratively, providing better and controllable continuum of message exposure.

### 3 Problem description

A revolution in TV advertising planning occurred in the beginning of 1980s when television audience measurement (TAM) started to be performed using electronic devices called people-meters [Rust and Eechambadi 1989]. In countries where TAM systems exist, such devices were installed in statistically significant number of homes so the data they provide can be considered reliable. The main information they collect are *who is watching what channel at what time*. Using these data in combination with TV program for each channel, it is possible to generate parameters that can be very useful in analysis of the audience and design of media plan.

Viewers are people in whose homes people-meters are installed. For each person it is known the characteristics that enable to be determined whether he/she belongs to the target audience or not. For each time slot it is known: to which TV channel it belongs, at what day/time it is aired and what is the cost of inserting TVC in it.

On the basis of raw data obtained by TAM, some parameters, widely used in formulation of media plans, can be calculated [Katz 2003, Schimp 2007]. A rating point represents one percent of a designated group or an entire population that is exposed to a particular advertising vehicle such as TV program. There are two main parameters based on rating points:

- Gross rating points (GRPs) are rating points for all viewers.
- Target rating points (TRPs) are rating points for a target audience.

In this paper, the acronym NE will be used for the number of exposures of an individual target viewer to TVC. Determining the proper NE has been the subject of many marketing studies. Researches in advertising have shown that the number of exposure must exceed some level, so called communication threshold, in order to form a potential consumer's desire for advertised product or service [Cannon et al. 2002]. Otherwise, the viewer can be considered underexposed. On the other hand, high level of exposure, i.e. overexposure would ultimately result in saturation and negative reactions on the message [Belch 1982]. In that case desire decreases and even aversion toward the product can appear.

The maximal efficiency of an advertising campaign will be achieved if the number of viewers exposed a proper number of times to TVC is maximized. For example, let a marketing study shows that NE to TVC should be between 3 and 5 (Figure 3.1). Advertising agencies usually make media plans based on TRPs, which are calculated as average on the target audience level. As a consequence, the situation presented in Figure 3.1(a) can appear: although the average NE is proper, there can be many viewers who were under or over exposed. If we want NE to be in the certain interval for as many viewers as possible, we have to optimize media plan using individual viewers data collected by TAM systems. Result of such optimization is presented in Figure 3.1(b).

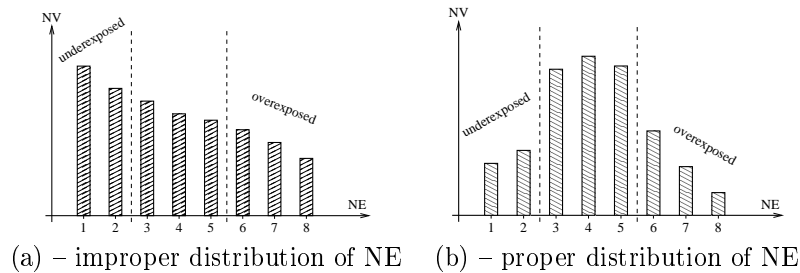


Fig. 3.1: Audience exposed to a certain number of messages

The primary goal of media plan is to provide proper NE to TVC for as many target viewers as possible. Important thing is that exposures occur continuously during entire period of campaign in accordance to chosen advertising strategy. We also control GRPs for each day of campaign. In addition, total cost of advertising campaign has to be as small as possible.

Having in mind the goals that should be accomplished by TV advertising campaign and available data, the goal programming appears to be the most suitable tool for modeling and generating media plans.

## 4 Mathematical model

In order to achieve the first goal we minimize the difference between targeted (wished, proper) and estimated NE for each target viewer.

Instead of observing the entire campaign period as a whole, we divide it into

sub-periods and we optimize media plan for each of them sequentially. This approach has two main advantages. First, we significantly reduce computational complexity of the problem. The second advantage of campaign period partitioning is that in this way we are able to control the dynamic of campaign and adjust it to our advertising strategy. The model we developed supports the following controls:

1. Control over NE for each sub-period – if a viewer was underexposed to TVC in one sub-period we will favor him in the next sub-period. On the other hand, if a viewer was overexposed, the targeted NE for him will be reduced in the next sub-period. This is a kind of diversification mechanism that prevents a certain group of viewers to be neglected while others are overexposed.
2. The regularity of viewing – if according to the previous rule for one target viewer NE is reduced, we determine what is the minimal NE he should have in order to preserve regularity of TVC consumption.
3. The budget control – we minimize cost, but if it is smaller than the budget for a certain sub-period, it is possible to use it for next sub-periods, completely or partially.

In order to solve this multi-objective optimization problem and to enable tuning the mentioned parameters a mixed integer goal programming mathematical model is developed.

We use the following notation:

### Sets

$Q$  set of all viewers covered by TAM;

$P$  set of target viewers covered by TAM;

$D$  set of days of campaign;

$E$  set of time slots during campaign;

$N = \{1, 2, \dots, n\}$  set of sub-periods;

$D_k$  set of days in  $k$ -th sub-period,  $k \in N$ , such that  $\bigcup_{k \in N} D_k = D$ ,  $D_r \cap D_s = \emptyset$ ,  
 $\forall r, s \in N, r \neq s$ ;

$E_l$  set of time slots in  $l$ -th day,  $l \in D$ , such that  $\bigcup_{l \in D} E_l = E$ ,  $E_r \cap E_s = \emptyset$ ,  
 $\forall r, s \in D, r \neq s$ ;

### Parameters

$p_{ij}$  expected NE of  $i$ -th time slot by  $j$ -th target viewer,  $i \in E$ ,  $j \in P$ ;

$gp_i$  GRP of  $i$ -th time slot,  $i \in E$ ;

$gpl_l, gpu_l$  lower and upper bounds of GRP for  $l$ -th day;

$c_i$  cost of inserting a TVC in  $i$ -th time slot,  $i \in E$ ;

$b_k$  budget for  $k$ -th sub-period;

$rb$  part of the surplus of budget from the previous sub-period that can be used in  $k$ -th sub-period,  $rb \in [0, 1]$  (typically 0 or 1);

$f q_k$  cumulative targeted NE in  $k$ -th sub-period ( $0 < f q_1 < f q_2 < \dots < f q_n$ );

$m f q_k$  minimal NE in  $k$ -th sub-period that should be reached even if a viewer was overexposed to TVC in the previous sub-period.

Among the parameters mentioned above, some are given by the advertiser and advertising agency while others are calculated upon the raw data obtained by TAM. Parameters given by the advertiser depend on advertising strategy and they are determined by marketing experts. Data from TAM systems are organized in tables and can be grouped for certain periods – cycles in which TV program is organized, typically for weeks. Elements of the tables  $a_{ij}^q \in \{0, 1\}$  have the following meaning: If viewer  $j$  saw time slot  $i$  in  $q$ -th period (week) then  $a_{ij}^q = 1$ ; otherwise  $a_{ij}^q = 0$ . Collecting such data for a longer period, e.g.  $m$  weeks, we can generate probabilities that viewer  $j$  will see time slot  $i$ :

$$p_{ij} = \frac{1}{m} \sum_{q=1}^m a_{ij}^q$$

Parameter  $p_{ij}$  also represents expected number of times that viewer  $j$  will see the time slot  $i$ . Another calculated parameter is GRP. For each time slot  $i \in E$  it is calculated upon raw input data using formula:

$$gp_i = \frac{100}{m |Q|} \sum_{q=1}^m \sum_{j \in Q} a_{ij}^q$$

Parameters  $c_i$ ,  $i \in E$  are determined by advertising agency and they are dependent on several other parameters, typically on GRP of  $i$ -th time slot, reputation of a particular TV show, prime time coefficient, etc.

### Variables

$x_i$  binary decision variables – denote whether the TVC will be inserted in  $i$ -th time slot ( $x_i = 1$ ) or not ( $x_i = 0$ ),  $i \in E$ ;

$dn_j^k, dp_j^k$  negative and positive deviation variables for NE – under-achievement and over-achievement of targeted NE of  $j$ -th target viewer in  $k$ -th sub-period,  $j \in P$ ,  $k \in N$ ;

$dgn_l, dgp_l$  negative and positive deviation variables for lower and upper bound, respectively, on GRPs in  $l$ -th day,  $l \in D$ .

#### Parameters obtained after solving $(k - 1)$ th problem

$FQ_j^{k-1}$  expected NE of  $j$ -th viewer during first  $k - 1$  sub-periods,  $FQ_j^0 = 0$ ,  
 $FQ_j^{k-1} = \sum_{q=1}^{k-1} \sum_{i \in E_q} p_{ij} x_i, \forall j \in P$ ;

$dn_j^{k-1}$  optimal value of deviation variable of problem  $k - 1$ ,  $dn_j^0 = 0, \forall j \in P$ ;

$br_{k-1}$  surplus of the budget after  $(k - 1)$ th sub-period,  $br_0 = 0$ ,  $br_{k-1} = b_{k-1} + rb \cdot br_{k-2} - \sum_{i \in E_{k-1}} c_i x_i$ .

#### Mathematical model for $k$ th sub-period

$$(\min) f_k = M_1 \sum_{j \in P} (dn_j^k + dp_j^k) + M_2 \sum_{j \in P} dn_j^{k-1} dn_j^k + M_3 \sum_{l \in D_k} (dgn_l + dgp_l) + \sum_{l \in D_k} \sum_{i \in E_l} c_i x_i$$

subject to

$$\begin{aligned} \sum_{l \in D_k} \sum_{i \in E_l} c_i x_i &\leq b_k + rb \cdot br_{k-1} \\ \sum_{i \in E_k} p_{ij} x_i + dn_j^k - dp_j^k &= \max \{fq_k - FQ_j^{k-1}, m f q_k\}, \quad j \in P \\ \sum_{i \in E_l} gp_i x_i + dgn_l &\geq gpl_l, \quad l \in D_k \\ \sum_{i \in E_l} gp_i x_i - dgp_l &\leq gpl_l, \quad l \in D_k \\ x_i &\in \{0, 1\}, \quad i \in E_l, l \in D_k \\ dn_j^k, dp_j^k &\geq 0, \quad j \in P \\ dgn_l, dgp_l &\geq 0, \quad l \in D_k \end{aligned} \tag{4.1}$$

Final solution is obtained by solving the sequence of the problems presented by Model (4.1) for  $k = 1, 2, \dots, n$ .

Mathematical model (4.1) represents a goal programming model for the multi-objective problem described above. The terms of the goal function have the following meaning:

- $M_1 \sum_{j \in P} (dn_j^k + dp_j^k)$  – penalization of deviations from the targeted NE in  $k$ -th sub-period.
- $M_2 \sum_{j \in P} dn_j^{k-1} dn_j^k$  – favor of viewers underexposed in  $(k - 1)$ th sub-period ( $dn_j^{k-1} > 0$ ).



- $M_3 \sum_{l \in D_k} (dgn_l + dgp_l)$  – penalization of deviations for bounds on daily GRP in  $k$ -th sub-period.
- $\sum_{l \in D_k} \sum_{i \in E_l} c_i x_i$  – costs for  $k$ -th sub-period.

When condition  $M_1 \gg M_2 \gg M_3 \gg \max_{1 \leq k \leq n} \{b_k\}$  holds, criteria are optimized lexicographically.

The first constraint provides that the costs for  $k$ -th sub-period do not exceed the available budget planned for that sub-period increased by surplus from the previous sub-period. The second constraint provides control over NE in  $k$ th sub-period and regularity of viewing. The right hand side will have a different value for cases when a viewer was underexposed or overexposed in previous sub-periods. Third and fourth constraint determine lower and upper bounds on daily GRP for entire audience.

## 5 Solving method and experiments

In order to validate our approach, we compared the expected quality of media plan obtained by Model (4.1) to an existing real life media plan. We had access to a media plan that was generated by an advertising agency for advertising a banking service. The duration of the campaign was 23 days, and we had all the data used by agency: list of possible time slots with TRPs and GRPs for each of them as well as costs of inserting TVCs in them and total budget. However, we were not able to get real life TAM data for that period. Instead, we generated them randomly, so that they correspond to the given parameters.

For experiments, a set of 15 different TAM tables were randomly generated each of them consisting of 1000 target viewers and 1670 time slots. Values  $a_{ij}$  were randomly generated in such a way that TRPs calculated upon them remain the same as original values. The campaign period of 23 days was divided into 6 sub-periods and targeted NE for each sub-period was 1, i.e. the targeted NE for entire campaign was 6. We adopted this value because that was the average result of media plan generated by advertising agency.

Our intention was to preserve the same campaign strategy and dynamics but still having better expected media plan efficiency in sense of criteria stated in Section 3. The total budget was allocated to sub-periods in such a way that it reflects the real life media plan dynamics.

For modeling and solving we used GNU Linear Programming Kit (GLPK), an open source software for solving linear and mixed integer mathematical programming problems.

Preliminary experiments have shown that it is not possible to solve real-life size problems to optimality in reasonable time. We used GLPK's implementation of Branch and Cut algorithm with limited execution time. In that way we were able to obtain an approximate solutions in reasonable time.

The comparison of the results of advertising agency and average results after 15 optimizations is given in Table 1. Remember that the primary goal of TV advertising plan is to provide proper NE to TVC for as many target viewers

Tab. 1: Average improvement of media plan quality

	NE 5-7	NE 4-8	Costs
Advertising agency	22.37%	35.93%	122688
Model (4.1)	36.75%	57.39%	77891
Improvement	64.28%	59.72%	36.51%

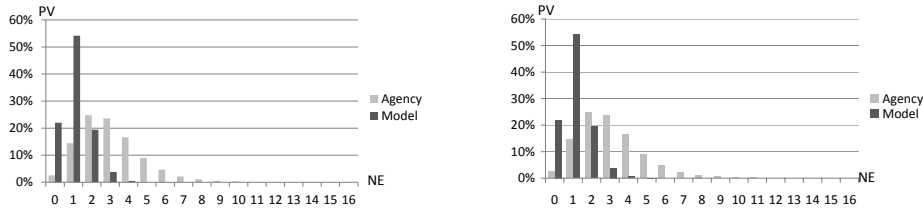


Fig. 5.1: Average change of NE through sub-periods

as possible. Since the targeted NE was 6, intervals  $6 \pm 1$  and  $6 \pm 2$  can be considered as proper. Values for NE in Table 1 represent percent of target viewers who saw TVC corresponding number of times. Improvements for both observed intervals are significant. Also important issue is dynamics of growth of NE through sub-periods. Distributions of NE after each of 6 sub-periods are presented in Figure 5.1 (from (a) through (f)) for both media plans. The peak of distribution for our media plan moves continuously in accordance with given cumulative target NE (1 per sub-period). On the other side, the distribution of NE for media plan generated by agency moves irregularly with tendency to make many overexposed viewers.

It is important to notice that the improvement of media plan quality is achieved with considerably smaller costs. This is consequence of reducing number of overexposed viewers (see Figure 5.1(f)).

## 6 Conclusion

Psychological and marketing researches have shown that proper exposure to TVC produces viewer's desire to consume the advertised product. A media plan which makes that as many target viewers as possible are exposed to TVC proper number of times with minimal possible costs is considered efficient.

We have presented a new approach to optimization of efficiency of media plan for TV advertising. This approach relies on raw data obtained by TAM systems. Using these data and iteratively solving goal programming problems highly efficient media plans can be obtained. The mathematical model we presented is flexible enough to enable modeling different kinds of advertising strategies.

Numerical experiments have shown that standard tools for solving mixed integer problems can be used for this problem to obtain near-optimal solutions. Based on real life data (time slots and their GRPs and TRPs), statistically

relevant number of TAM tables was randomly generated. In experiments we obtained more efficient media plans than one obtained by advertising agency regarding both number of viewers exposed to TVC proper number of times and costs.

Difficulties in using this approach can appear because of lack of raw TAM data. Our experience has shown that companies that collect such data are not willing to share them with advertising agencies. Perhaps, seeing the possibility to gain mutual advantages from optimization systems that use those data, they will become more opened for cooperation.

The next step in research in this area is to obtain real-life raw TAM data and to compare solutions obtained by this approach and standard solutions offered by advertising agencies.

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